

Solving Equations by Factoring - Using the Zero Product Property

The Zero Product Property:

If $xy = 0$, then either $x = 0$ or $y = 0$.

Use the zero product property to solve the following equations.

Ex 1: $x(x-1) = 0$

$x = 0$ or $x - 1 = 0$

$x = 1$

$x = 0, 1$

Ex 2: $(x-5)(x+2) = 0$

$x - 5 = 0$ or $x + 2 = 0$

$x = 5$ or $x = -2$

$x = -2, 5$

Ex 3: $5x(x-4) = 0$

$5x = 0$ or $x - 4 = 0$

$x = 0$ or $x = 4$

$x = 0, 4$

of factors indicates
of solutions!

If the polynomial is not factored, factor the polynomial before trying to use the zero product property to solve.

Ex 4: $2x^2 - 10x = 0$

$2x(x-5) = 0$

$2x = 0$ or $x - 5 = 0$

$x = 0$ or $x = 5$

$x = 0, 5$

Ex 5: $x^2 - 36 = 0$

$x^2 + 0x - 36 = 0$

$(x^2 - 6x) + (6x - 36) = 0$

$x(x-6) + 6(x-6) = 0$

$(x-6)(x+6) = 0$

$x - 6 = 0$ or $x + 6 = 0$

$x = 6$ or $x = -6$

$x = \pm 6$

$$\text{Ex 6: } 2x^2 - 7x + 3 = 0$$

$$(2x^2 - 6x) + (-x + 3) = 0$$

$$2x(x-3) + (-1)(x-3) = 0$$

$$(x-3)(2x-1) = 0$$

$$x-3=0 \quad \text{or} \quad 2x-1=0$$

$$x=3$$

$$2x=1$$

$$x = \frac{1}{2}$$

$$x = \frac{1}{2}, 3$$

$$\begin{array}{r|l} x(6) & +(-7) \\ \hline -6(-1) & \checkmark \end{array}$$

$$\text{Ex 7: } x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x[(x^2 - 2x) + (x - 2)] = 0$$

$$x[x(x-2) + 1(x-2)] = 0$$

$$x(x-2)(x+1) = 0$$

$$x=0 \quad \text{or} \quad x-2=0 \quad \text{or} \quad x+1=0$$

$$x=2$$

$$x=-1$$

$$x = -1, 0, 2$$

$$\begin{array}{r|l} x(-2) & +(-1) \\ \hline -2(1) & \checkmark \end{array}$$

$$\text{Ex 8: } 6x^3 - 46x^2 - 16x = 0$$

$$2x(3x^2 - 23x - 8) = 0$$

$$2x[(3x^2 - 24x) + (x - 8)] = 0$$

$$2x[3x(x-8) + 1(x-8)] = 0$$

$$2x(x-8)(3x+1) = 0$$

$$2x = 0 \text{ or } x - 8 = 0 \text{ or } 3x + 1 = 0$$

$$x = 0$$

$$x = 8$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$x = -\frac{1}{3}, 0, 8$$

If the polynomial is not "set equal to zero", get all of the terms on one side of the equation first. Then factor the polynomial before trying to use the zero product property to solve.

$$\text{Ex 9: } x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x^2 - 5x) + (2x - 10) = 0$$

$$x(x-5) + 2(x-5) = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0 \text{ or } x+2=0$$

$$x = -2, 5$$

$$\text{Ex 10: } 18 - 3x = x^2$$

$$0 = x^2 + 3x - 18$$

$$0 = (x^2 - 3x) + (6x - 18)$$

$$0 = x(x-3) + 6(x-3)$$

$$0 = (x-3)(x+6)$$

$$x-3=0 \text{ or } x+6=0$$

$$x=3$$

$$x=-6$$

$$x = -6, 3$$

$$\text{Ex 11: } 8x^2 + 5x - 2 = 4x^2 + 12x$$

$$4x^2 - 7x - 2 = 0$$

$$(4x^2 - 8x) + (x - 2) = 0$$

$$4x(x - 2) + 1(x - 2) = 0$$

$$(x - 2)(4x + 1) = 0$$

$$x - 2 = 0 \text{ or } 4x + 1 = 0$$

$$x = 2 \quad 4x = -1$$

$$x = -\frac{1}{4}$$

$$\boxed{x = -\frac{1}{4}, 2}$$

$$\text{Ex 12: } (x + 3)^2 = -2(x^2 - 2x - 7)$$

$$(x + 3)(x + 3) = -2x^2 + 4x + 14$$

$$x^2 + 3x + 3x + 9 = -2x^2 + 4x + 14$$

$$\begin{array}{ccccccc} +2x^2 & -4x & -14 & +2x^2 & -4x & -14 \\ \hline \end{array}$$

$$3x^2 + 2x - 5 = 0$$

$$(3x^2 - 3x) + (5x - 5) = 0$$

$$3x(x - 1) + 5(x - 1) = 0$$

$$(x - 1)(3x + 5) = 0$$

$$x - 1 = 0 \text{ or } 3x + 5 = 0$$

$$x = 1$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

$$\begin{array}{l} x(-15) \quad | \quad + (2) \\ \hline 5(-3) \quad | \quad \checkmark \end{array}$$

$$\boxed{x = -\frac{5}{3}, 1}$$